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Measurement system of warranty returns.

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MEASUREMENT SYSTEM OF

WARRANTY RETURNS

by

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TABLE OF CONTENTS

	Page
Abstract	1
Chapter	
I. Introduction	3
II. Construction of Return Rate Model	11
III. Model Experimentation	26
Conclusions	40
For Future Investigation	42
Appendix A. Data Set #1	43
Appendix B. Data Set #2	44
Appendix C. Data Set #3	45
Appendix D. Samples of Sales Weighting Factors . . f(w) for Data Set #1	46
Appendix E. Samples of Sales Weighting Factors . . for Data Set #3	47
Appendix F. The "Vita"	48
Bibliography	49

LIST OF FIGURES

Figure		Page
a	Representation of one month of sales	7
b	Representation of one month of sales and their related returns	8
c	Representation of one month of sales and their related returns each month throughout the warranty period	9
d	Representation of month one sales and their related returns each month throughout the warranty period	11
e	Representation of month two sales and their related returns each month throughout the warranty period	12
f	Representation of month three sales and their related returns each month throughout the warranty period	14
g	Composite of figure d, e and f related to the same time frame	16
h	Hypothetical representation of sales factors f(w)	18

LIST OF TABLES

Table		Page
1	Theoretical representation of returns, sales and warranty return rate formulas	17
2	Theoretical representation of returns and sales in both the conventional form and Mandel form	24
3	Data Set #1	34
4	Data Set #2	35
5	Data Set #3	36
6	Data Set #2 with cumulative sales and returns .	43
7	Data Set #2 with cumulative sales and returns .	44
8	Data Set #3 with cumulative sales and returns .	45

LIST OF LINE GRAPHS

Graph		Page
1	Return rate percentages, data set 1, using weighted sales and nonweighted sales	37
2	Return rate percentages, data set 2, using weighted sales and nonweighted sales	38
3	Return rate percentages, data set 3, using weighted sales and nonweighted sales	39
4	Samples of f(w) which minimized the variance of data set 1	46
5	Samples of f(w) which minimized the variance of data set 3	47

ABSTRACT

It is difficult to measure the warranty return rate of a mass produced product which is sold in almost every department store in the country. It is even more difficult to measure the warranty return rate of the same product before the warranty period of the initial sales units have expired.

The return rate is a useful index. It gages the reliability success of new product introductions, it provides a historical picture of product field performance, and it is useful for measuring the impact of reliability improvement changes.

The warranty return rate is a ratio between the number of products returned during the warranty period as compared to the number of products sold. On the surface, the ratio appears to be a simple one, but in reality it is not. This month's sales will effectively contribute warranty returns for the duration of the warranty period. For example, if the warranty period were twelve months, it is conceivable that twelve months' worth of units sold would effect a twenty-four month period of warranty returns. To avoid the natural reaction of incorrectly stating the return rate, a mathematical scheme is necessary to adjust the data to obtain meaningful results.

This thesis generates a generally applicable mathematical model using the Beta distribution, such that realistic product return

rates can be calculated.

The model is capable of delivering meaningful return rates before the end of the warranty period from the very first month of new product sales. Also, the model is compatible with systematic computer calculations so that an effective return reporting system could be implemented using this system.

CHAPTER I

INTRODUCTION

A reliable product is one that can be counted on to perform the function it is designed to perform when called upon. To put it another way, "when you press the button it works".

The purpose of this paper is to develop a means of measuring, in a sense, the reliability of a product. The problem of measurement is rather unique because the products are relatively low cost items. They are mass produced and are sold in almost every retail department store in the United States. Product returns are handled through fifty company owned service centers and approximately three times as many independent servicers throughout the nation. The measurement we are after consists of a ratio between the number of units, from a given product, which are returned during the warranty period, in comparison to the number of products which are sold.

The uniqueness of the problem stems from the fact that product serialization does not exist and therefore product traceability for each production unit does not exist from the date of manufacture, throughout warehouse storage, throughout being carried in a retailers inventory, through the date of sale, and finally, if it does happen, the date the product is returned for warranty servicing. If this information were available the measurement of product returns would be rather simple and straight foreward.

For example serialization information, of this nature, is available in the automotive industry and other industries which manufacture high ticket items or high priced products.

Product reliability is inherent in product technology when designed, built and inspected under optimal conditions. Ideally, a company can life test its products and accurately evaluate its reliability and percentage of warranty returns. In practice, however, there are some factors which introduce inaccuracy and uncertainty to such life test experiments, such as:

1. The company may not be able to duplicate field conditions accurately, especially with accelerated life tests.

2. Periodic variation in material and processing contributes to variations in actual product reliability.

3. Poor workmanship cannot always be detected in time, or be controlled and corrected.

4. Precise prediction of small failure percentages may require large samples. Life testing of such samples for a long period of time may be beyond the company's capability.

5. Even if the company conducts short, accelerated life test experiments, on a continuous basis to simulate a long warranty period, the results may be too late to affect current production.

Usually a combination of these reasons may prevent the company from establishing a continuous accurate reliability prediction program. In most cases reliability can be controlled only within practical limits.

In addition other reasons for product returns occur. Ineffective packaging design can lead to damaged products when received by the consumer. Even though, when they left the factory they were in top condition. A product user may not accept the performance of the product to be within their liking and therefore return the product. So in effect, many items influence the product returns during the warranty period.

If the common life characteristic curve were looked at, the two zones of concern which make up the warranty period include the infant mortality period and the normal operating period. The first period is termed the infant mortality period and is caused by early failure of weak components due principally to "assignable causes" of non random nature. This period is typified by a fairly high failure rate which drops off rapidly in the first few product usages. The second period is typified by a fairly constant rate of failure. Failures occur in a random manner associated with a constant cause system. There exists also a third period which normally does not affect the warranty returns but is most typical of the wearout phase or out of warranty servicing. The third period is termed the wearout period in which the failure rate starts to rise rapidly as the number of survivors approach zero until all units have failed and no more are "left to die". [1]

The object is to measure the product return rate prior to the end of the warranty period from the first units sold. The following set of conditions exist for each product model

1. The monthly product sales to retailers, from the warehouses throughout the nation, are accurately known.
2. The monthly product returns for each product are accurately accounted at each company owned and company franchised service center.
3. The warranty period is twelve months. So for a given month of sales, the product returns could continue to flow back to the service centers for twelve months subsequent to the sale.
4. No serialization exists in the monthly sales and return data. In other words the product return is not readily traceable to a given period of product sales.

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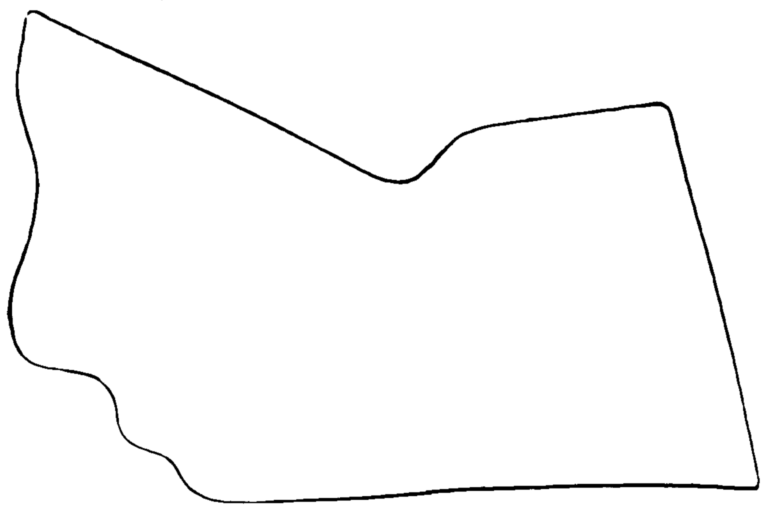
1 See Igor Bazovsky, Reliability Theory and Practice, (Englewood Cliffs: Prentice-Hall, Inc.) 1963 pp. 32-35.

A set of factors will be experimentally derived, using the system developed later in the report, to provide a best fit model so the number of failures, which are going to occur, during each month of the warranty period subsequent to a given month of product sales can be obtained.

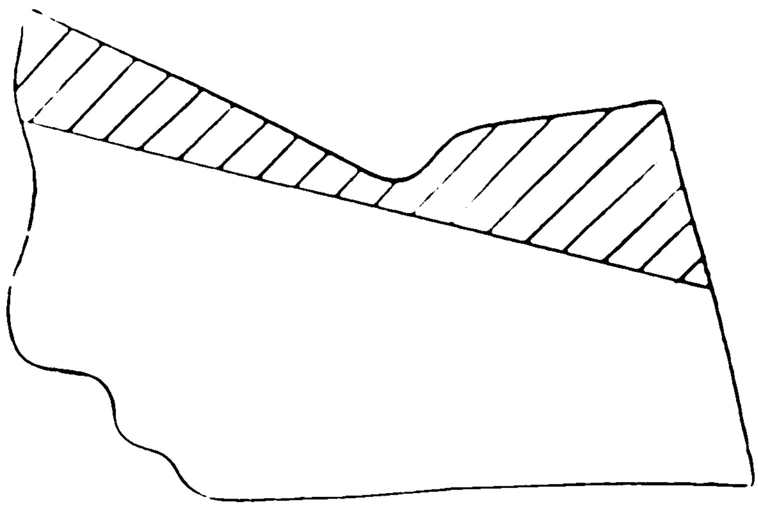
Pictorially

(a) Figure (a)



The area within the solid perimeter represents one month of product sales.

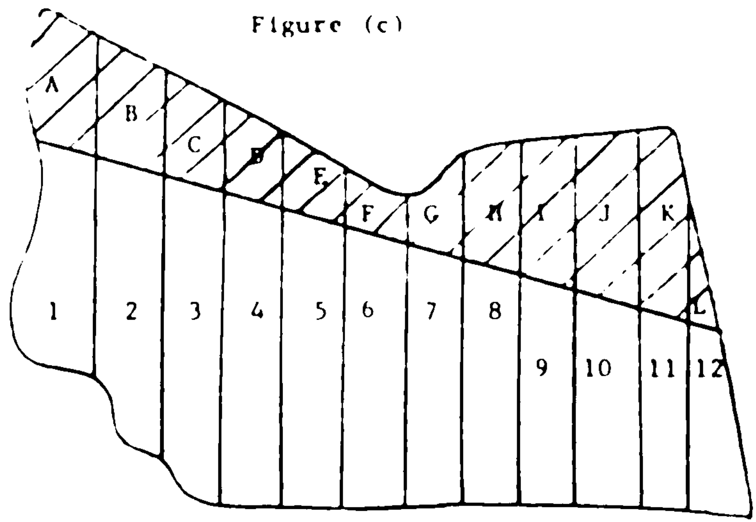
(b) Figure (b)



The area within the perimeter is divided into two sections. The cross hatched area represents the number of product returns generated during the warranty period from one given month of sales, the non cross hatched portion of the area represents units which were not returned during the warranty period.

(c) Figure (b) is now redrawn and the area inside the perimeter of the figure is sliced into twelve not necesarily equal slices. Each slice represents one month of the warranty period.

Figure (c)



The letters label the plots of product returns which have occurred during the warranty period. Each letter represents the number of returns during each month of the warranty period.

The numbers label the plots of product sales which are not returned during the warranty period.

Plot (1 + A) can be interpreted therefore as (1 + A) sales which contributed (A) returns during the first month of the warranty period having a return rate equal to $\left\{ \frac{A}{1 + A} \right\}$.

Plot (2 + B) can be interpreted also as (2 + B) sales which contributed (B) returns during the second month of the warranty period. The return rate after the second month of the warranty period would equal $\left\{ \frac{A + B}{1 + A + 2 + B} \right\}$. This same analysis can be carried out until the entire area within the perimeter is covered and the Final Failure Rate from one month of Sales would equal.

$$\frac{A+B+C+D+E+F+G+H+I+J+K+L}{1+A+2+B+3+C+4+D+5+E+6+F+7+G+8+H+9+I+10+J+11+K+12+L}$$

The subsequent chapters will develop for a given product the theoretical model which will slice the area of sales for each given month proportionally, not necessarily equal, such that return rates can be estimated prior to the end of the warranty period. As can be seen from figure (b), if one waited until the end of the warranty period the return rate could be easily derived. It would equal the Products Returned (cross hatched area) divided by the sales (total area enclosed by the perimeter).

Also another chapter is devoted to actual experimentation with the model using real data inorder to select the best slicing technique to give realistic estimates of the product return rate. Finally the report is terminated with conclusions and recommend for further investigation.

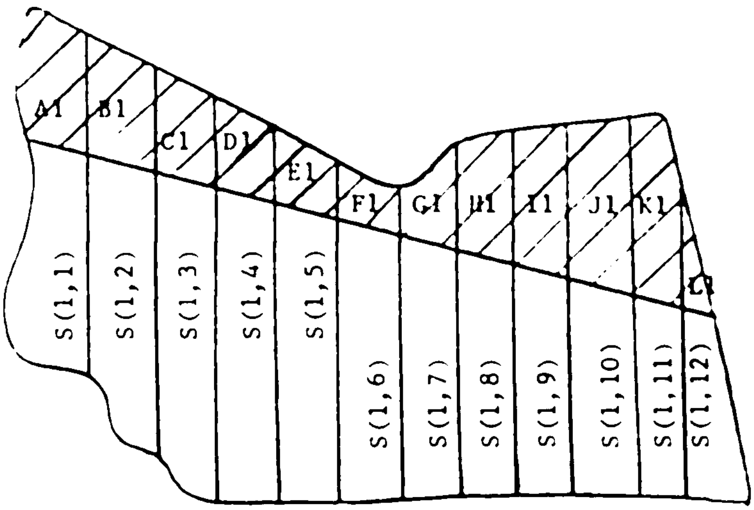
CHAPTER 2

CONSTRUCTION OF RETURN RATE MODEL

This chapter details the construction of the mathematical model which will compute the return rate.

Construction of the model will begin by redrawing figure (c) from Chapter 1. The only difference will be the identification sequence of the different areas within the perimeter.

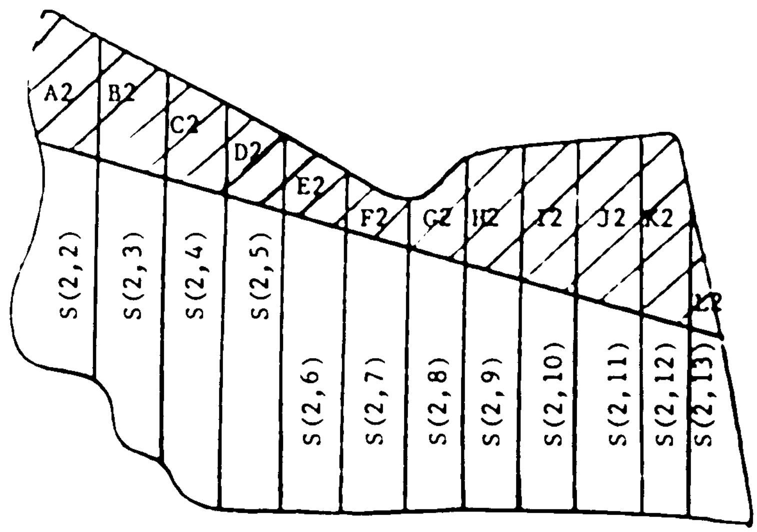
Figure (d)



The area within figure (d) represents the total number of sales during month one. The area is sliced into twelve, not necessarily equal slices. Each slice represents one month of the warranty period and is labeled S(1,1), S(1,2), S(1,3) -,-,-,-, thru S(1,12). The sum of the slices S(1,1) + S(1,2) + S(1,3) +---+ S(1,12) equals the total number of sales from month one. The portion of each slice labled A1, B1, C1, -,-,-, thru L1 represent the warranty returns through the warranty period from month one sales. Areas A1 + B1 + C1 + ---- + L1 equal the total number of warranty returns from the first month of sales. Slice S(1,1) represent the portion of sales from month one which generate the warranty returns A1 during month one. Slice S(1,2) represents the portion of sales from month one which generates warranty returns B1 during month two after the sale. Slice S(1,3) represents the portion of sales from month one which generate warranty returns C1 in month three after the sale. Each slice repeats until the end of the warranty period when slice S(1,12) represents the final portion of sales from month one which generate warranty product returns L1 during month twelve after the sale.

Figure (e) is basically the same as figure (d) except it now represents the product sales that have taken place during month two.

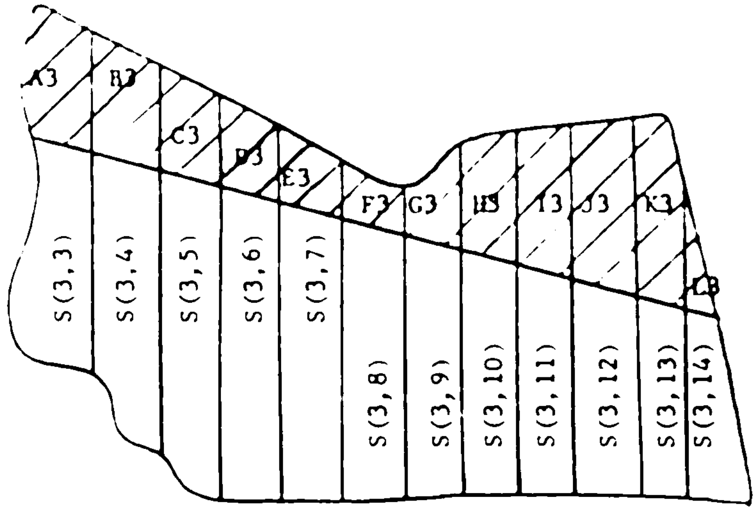
Figure (e)



The area within figure (e) represents the total number of sales during month two. The area is sliced into twelve, not necessarily equal slices. Each slice represents one month of the warranty period and is labled S(2,2), (S(2,3), S(2,4) ,-,-,-, S(2,13). The sum of the slices S(2,2) + S(2,3) +---+ S(2,12)equals the total number of sales from month two. The portion of each slice labled A2, B2, C2, -,-,-, thru L2 represent the warranty product returns through the warranty period from month two sales. Areas A2 + B2 + C2 + --- + L2 equal the total number of warranty returns from the second month of sales. Slice S(2,2) represents the portion of sales from month two which generate the warranty returns A2 during month one after the sale. Slice S(2,3) represents the portion of sales from month two which generates warranty returns B2 during the second month after the sale. Slice S(2,4) represents the por- of sales from month two which generate product returns C2 during month three after the sale. Each slice repeats until the end of the warranty period when slice S(2,13) represents the final portion of sales from month two which generates warranty returns L2 during month twelve after the sale.

Figure (f) is basically the same as figure (d) and (e) except it now represents the product sales that have taken place during month three.

Figure (f)



The area within the perimeter of figure (f) represents the total number of sales during month three. The area is sliced into twelve, not necessarily equal, slices. Each slice represents one month of the warranty period and is labled S(3,3) S(3,4), S(3,5) ,-,-, thru S(3,14). The sum of the slices S(3,3) + S(3,4) + S(3,5) +----+ S(3,14) equals the total number of sales from month three. The portion of each slice labled A3, B3, C3, -,-, thru L3 represent the warranty returns through the warranty period from month three sales. Areas A3 + B3 + C3 +---+ L3 equal the total number of warranty returns from the third month of sales. Slice S(3,3) represents the portion of sales from month three which generate the warranty returns A3 during month one after the sale. Slice S(3,4) represents the portion of sales from month three which generate warranty returns B3 during the second month after the sale. Each slice repeats until the end of the warranty period when slice S(3,14) represents the final portion of sales from month three which generates warranty returns L3 during month twelve after the sale.

Figure (g) represents a composit of figures (d), (e) and (f), related to the same time frame. From this representation the mathematical model can begin to be constructed.

The return rate calculation consists of the sum of the returns divided by the sum of the sales. Using figure (g), a table will be constructed to aid development of the return rate equation. The table will consist of three parts for each month shown:

1) Applicable Returns
2) Applicable Sales
3) Return Rate Formula

Figure (g)

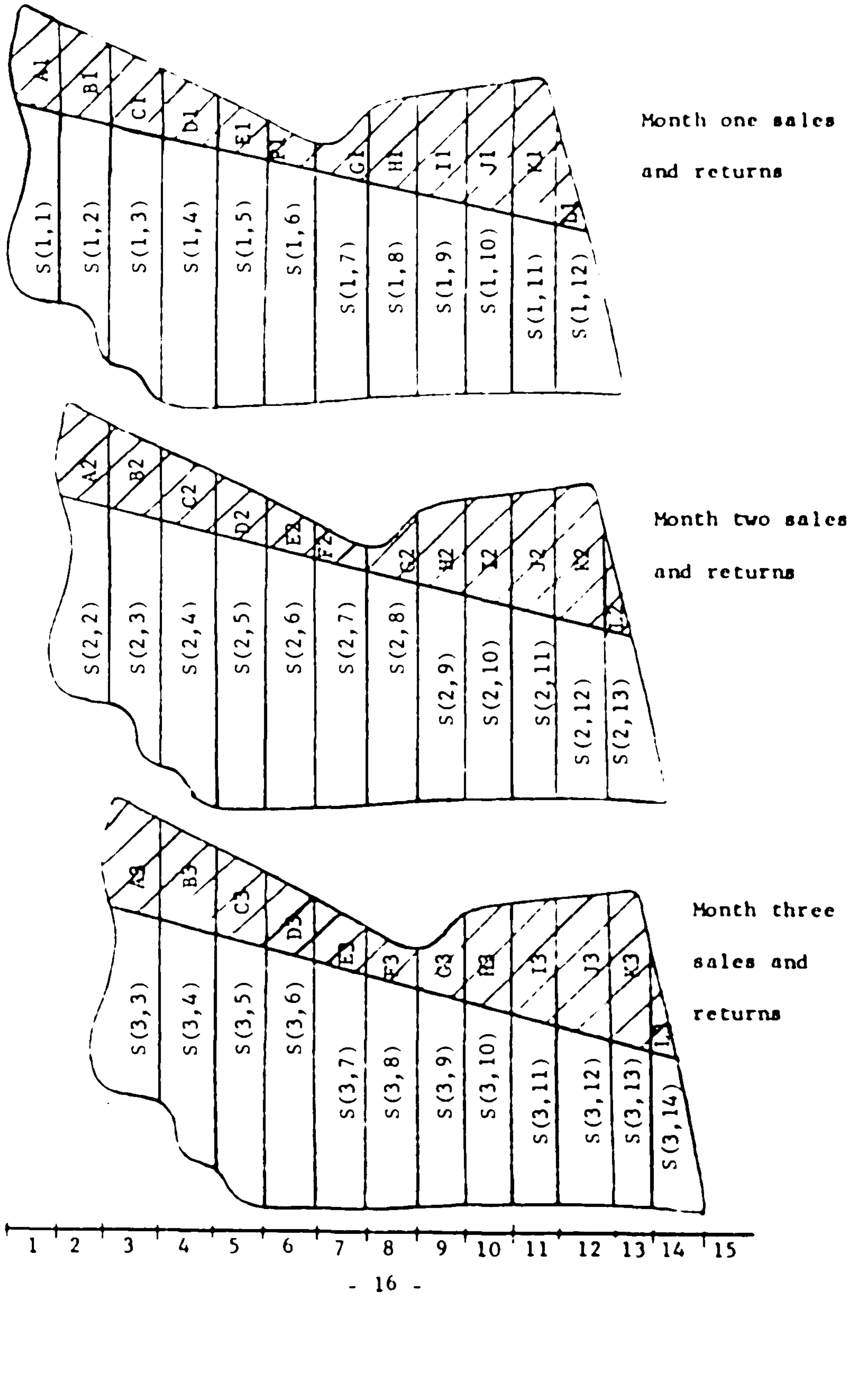

Month one sales and returns
Month two sales and returns
Month three sales and returns
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

TABLE 1

MONTH	RETURNS	SALES	WARRANTY RETURN RATE FORMULAS
1	A1	S(1,1)	$\frac{A1}{S(1,1)}$
2	A1+B1+A2	S(1,1)+S(1,2)+S(2,2)	$\frac{A1+B1+A2}{S(1,1)+S(1,2)+S(2,2)}$
3	A1+B1+C1+A2+ B2+A3	S(1,1)+S(1,2)+S(1,3)+ S(2,2)+S(2,3)+S(3,3)	$\frac{A1+B1+C1+A2+B2+A3}{S(1,1)+S(1,2)+S(1,3)+S(2,2)+S(2,3)+S(3,3)}$

Let A1 = R1, A2+B1 = R2, and let A3+B2+C1 = R3

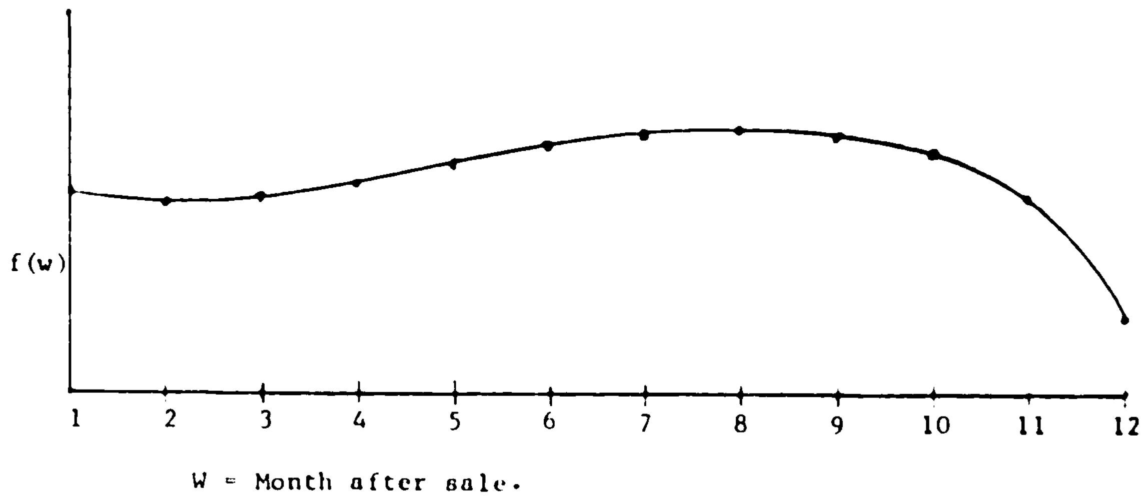
Rewriting Month 3 we have

3	R1+R2+R3	S(1,1)+S(1,2)+S(1,3)+ S(2,2)+S(2,3)+S(3,3)	$\frac{R1+R2+R3}{S(1,1)+S(1,2)+S(2,2)+S(1,3)+S(2,3)+S(3,3)}$
x	$\sum_{y=1}^{x} R(y)$	$\sum_{y=1}^{x} \sum_{p=1}^{y} S(p,y)$	$\frac{\sum_{y=1}^{x} R(y)}{\sum_{y=1}^{x} \sum_{p=1}^{y} S(p,y)}$

Where p = the month of sale and y = the month of return

The next step is the design of the function f(w) which slices each of the areas shown in figures (d), (e), (f) into the number of slices which are equal to the number of months in the warranty period.

Figure (h) (Sales Factors)



The constraints on f(w) are the following:

a) $\sum_{W=1}^{12} f(w) = 1$

b) $f(w) = 0$ When $W < 1$

c) $f(w) = 0$ When $W > 12$

d) $f(w) \geq 0$ When $1 \leq W \leq 12$

Therefore, let $f(y-p+1) = f(w)$ then the following relationship will exist provided $f(w)$ possess the properties described in figure h.

$S(p,y) = S(p)*f(y-p+1)$

p = the month of sale

y = the month of return

Using this the following general expression could then be written for returns, sales and return rate for any month x after the initial product sale.

a) Cumulative Product returns thru month $X = \sum_{y=1}^{x} R(y)$

b) Cumulative Product Sales affecting the product returns thru month $x = \sum_{y=1}^{x} \sum_{p=1}^{y} S(p)*f(y-p+1)$

c) Warranty Return rate for month $x = \dfrac{\sum_{y=1}^{x} R(y)}{\sum_{y=1}^{x} \sum_{p=1}^{y} S(p)*f(y-p+1)}$

The preceeding equation has a striking similarity to the equation $Y = A + bX$. Where $A = 0$, b = the warranty return rate r,

$$Y = \sum_{y=1}^{x} R(y), \text{ and } X = \sum_{y=1}^{x} \sum_{p=1}^{y} S(p)*f(y-p+1).$$

Therefore it appears the solution of the equation could be obtained for r using the least squares method.[1]

Using the form Y = A + bX we can write
$$\sum_{y=1}^{x} R(y) = r \sum_{y=1}^{x} \sum_{p=1}^{y} \left[S(p)*f(y-p+1) \right] + E$$

So that the sum of squares of deviations from the true line is

$$S = \sum_{x=1}^{z} E(z)^2 = \sum_{x=1}^{z} \left[\sum_{y=1}^{x} R(y) - r \sum_{y=1}^{x} \sum_{p=1}^{y} \left[S(p)*f(y-p+1) \right] \right]^2$$

Let $M = \sum_{y=1}^{x} R(y)$

Let $T = \sum_{y=1}^{x} \sum_{p=1}^{y} \left[S(p)*f(y-p+1) \right]$

The equation could then be re-written as $S = \sum_{x=1}^{z} E(2)^2 = \sum_{x=1}^{z} \left[M-rT \right]^2$

---

[1] See N. R. Draper and H. Smith, Applied Regression Analysis (John Wiley and Sons, Inc., 1966), pp. 7-11.

r is determined by differentiating the above equation with respect to r and setting the results equal to zero.

$$\frac{\partial S}{\partial r} = -2 \sum_{x=1}^{z} T \left[M - rT \right] = 0$$

$$\sum_{x=1}^{z} T*M - r*\sum_{x=1}^{z} T^{2} = 0$$

$$r = \frac{\sum_{x=1}^{z} T*M}{\sum_{x=1}^{z} T^{2}}$$

$$r = \frac{\sum_{x=1}^{z} \left[\left[\sum_{y=1}^{x} R(y) \right] * \sum_{y=1}^{x} \sum_{p=1}^{y} \left[S(p)*f(y-p+1) \right] \right]}{\sum_{x=1}^{z} \left[\sum_{y=1}^{x} \sum_{p=1}^{y} \left[S(p)*f(y-p+1) \right]^{2} \right]}$$

This equation seems to be a good solution for r but according to Mandel[2] the error associated with each point in the equation development includes the errors associated with all previous points. Therefore, the least squares is the incorrect method of analyzing the type of cumulative data. He suggests a more correct method of handling data with cumulative errors. The following analysis illustrates his approach. The form of the developed

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2 See John Mandel, The Statistical Analysis of Experimental Data (John Wiley and Sons, Inc., 1964), pp. 295-303.

equation

$$\sum_{x=1}^{z}\left[\sum_{y=1}^{x} R(y)\right] - r*\sum_{x=1}^{z}\left[\sum_{y=1}^{x}\ \sum_{p=1}^{y}\left[S(p)*f(y-p+1)\right]\right] + r\sum_{x=1}^{z} E(x)$$

appears to be identical to

$$Y(i) = BX(i) + B\sum_{k=1}^{i} E(k).$$

According to Mandel it is seen at once that one of the basic assumptions of the "classical" case is violated· the errors corresponding to different points are not statistically independent, because the series of numbers

$E(1),\ E(1) + E(2),\ E(1) + E(2) + E(3),\ - - -$

are not independent, each one includes all previous ones. He develops a relationship for cumulative data such that the error terms are statistically independent. He converts the X data in the following manner

Let $L(i) = X(i) - X(i-1)$

and he converts the y data in the following manner

$$Z(i) = Y(i) - Y(i-1)$$

Applying the least squares calculations he obtains

$$B = \sum_{i} Z(i)/L(i)$$

and $$VB = \sum_{i}\left[d(i)^{2}/L(i)\right]/\left[(N-2)\sum_{i} L(i)\right]$$

Where the residual d(i) is defined as.

$d(i) = Z(i) - B*L(i)$

We can now apply the data in table 1 to the equations developed by Mandel. The $L(i)$ terms will relate to the Sales data and the $Z(i)$ term will relate to the returns data. Table 2 is a conversion of the data in table 1 using the Mandel approach.

TABLE 2

	CONVENTIONAL FORM		MANDEL FORM	
MONTH i	$Y(i)$ RETURNS	$X(i)$ SALES	RETURNS $Z_i = Y_i - Y_{i-1}$	SALES $L_i = X_i - X_{i-1}$
1	R1	S(1,1)	-	-
2	R1+R2	S(1,1)+S(1,2)+S(2,2)	R2	S(1,2)+S(2,2)
3	R1+R2+R3	S(1,1)+S(1,2)+S(2,2)+ S(1,3)+S(2,3)+S(3,3)	R3	S(1,3)+S(2,3)+S(3,3)
x	$\sum_{y=1}^{x} Ry$	$\sum_{y=1}^{x} \sum_{p=1}^{y} S(p,y)$	Rx	$\sum_{p=1}^{x} S(p,x)$
			$\sum_{i} Z(i) = \sum_{i=2}^{x} R(i)$	$\sum_{i} L(i) = \sum_{i=2}^{x} \sum_{p=1}^{i} S(p,i)$

$$\text{Mandel B} = \text{Return Rate } r(x) = \frac{\sum_{i=2}^{x} R(i)}{\sum_{i=2}^{x} \sum_{p=1}^{i} S(p,i)}$$

Therefore using Mandel's approach the warranty return rate cannot be computed after the first period of data but it could be calculated for subsequent data periods. For example

$$\text{Return rate period } (2) = r(2) = \frac{R(2)}{S(1,2) + S(2,2)}$$

$$\text{Return rate period } (3) = r(3) = \frac{R(2) + R(3)}{S(1,2)+S(2,2)+S(1,3)+S(2,3)+S(3,3)}$$

$$\text{Return rate period } (x) = r(x) = \frac{\sum_{i=2}^{x} R(i)}{\sum_{i=2}^{x} \sum_{p=1}^{i} S(p,i)}$$

Where $S(p,i) = S(p)*f(i-p+1)$ and the Variance of the return rate is -

$$Vr(x) = \frac{\sum_i d(i)^2/L(i)}{\left[(X-1)-2\right]*\sum_i L(i)} \qquad \text{Where } d(i) = z(i)-r(x)*L(i)$$

The following Chapter uses the developed equations for $r(x)$ and $Vr(x)$ along with a search routine using many shapes of $f(w)$ to find the best value of $r(x)$ which corresponds to the minimum variance $Vr(x)$ the best solution for Return Rate.

CHAPTER 3

MODEL EXPERIMENTATION

This Chapter deals with experimentation using the model developed in the previous Chapter.

$$\text{Return Rate Equation} = r(x) = \frac{\sum_{i=2}^{x} R(i)}{\sum_{i=2}^{x} \sum_{p=1}^{i} S(p,i)}$$

Where $S(p,i) = S(p)*f(i-p+1)$ and

$$\text{Variance Equation} = Vr(x) = \frac{\sum_{i=2}^{x} d(i)^2/L(i)}{\left[(x-1)-2\right] * \sum_{i=2}^{x} L(i)}$$

Where $d(i) = Z(i)-r(x)*L(i)$

A search was undertaken to find the most optimum values for the sales factors f(w) which minimizes the variance for the return rate Vr(x). The search strategy consisted of writing a computer program for evaluating different data sets each containing twenty-five paired values of product sales and returns to find the minimum Vr(x). This was accomplished by calculating r(x) and Vr(x) for all paired data periods, across a whole host of f(w) values. The resultant computer output data was manually searched to find the minimum Vr(x) and its corresponding r(x) and f(w) for each of the twenty-five periods represented by the data.

Where f(w) met the following constraints

$$\sum_{w=1}^{12} f(w) = 1$$

$f(w) = 0$ if $w < 1$

$f(w) = 0$ if $w > 12$

$f(w) > 0$ when $1 \leq w \leq 12$

The first approach was to develop individual sets of f(w) in a systematic fashion and evaluate the variance Vr(x) and return rate r(x). For example the initial f(w)'s that were tried took the form of the following:

a) f(w) = Constant for w=1 to 12

b) f(w) = Combination of constant and ramp for w = 1 to 12

c) f(w) = Binomial probability density function[1]

$$\frac{n!}{r!(n-r)!} * \left[p^{r} (1-p)^{n-r} \right]$$

Where n = 11, r = 11 and p has values from .01 to .99. None of the above values of f(w), however, enabled minimum values of Vr(x) because the actual product return patterns did not fit the above generated f(w)'s. Another model for f(w) was tried which enabled the values of f(w) to take on many shapes which were not achievable

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1 See Martin L. Shooman, <u>Probabilistic Reliability: An Engineering Approach</u> (McGraw-Hill, Inc., 1968) pp. 33-36.

in a, b and c above. The model which appeared most flexible and able to take on many shapes was the Beta distribution model.[2]

$$f(w) = \frac{(a + b - 1)!}{(a-1)!\ (b-1)!}\ t^{a-1}(1-t)^{b-1}$$

where $0 \leq t \leq 1$

$a,\ b > 0$

To obtain twelve equally spaced values of t one for each month let

$$t = \frac{W + 1/2}{13} = 1/13\ W + 1/26$$

Different values of a and b, when plugged into the above equation for f(w) yield an almost infinite number of shapes of f(w).

A sample calculation using the developed equations for return rate and variance of return rate are accomplished in the following manner: Assume

$f(1) = .0024987$

$f(2) = .0199032$

$f(3) = .0720632$

$f(4) = .1565510$

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2 See Gerald J. Hahn and Samuel S. Shapiro, Statistical Models in Engineering (John Wiley and Sons, Inc., 1968), pp. 91-97.

and

		Period(1)	Period(2)	Period(3)	Period(4)
Sales	S(x)	36,200	10,400	7,800	29,400
Returns	R(x)	3	48	173	372

S(p,i) is read, sales from Period p which contribute to the returns during month i after the initial sales

then

S(p,i)	=	S(p)*f(i-p+1)			
S(1,1)	=	S(1)*f(1)			
S(1,1)	=	36,200*	.0024987	=	90.138
S(1,2)	=	36,200*	.0199032	=	720.38
S(1,3)	=	36,200*	.0720632	=	2608.57
S(1,4)	=	36,200*	.1565510	=	5667.11
S(2,2)	=	10,400*	.0024987	=	25.896
S(2,3)	=	10,400*	.0199032	=	206.96
S(2,4)	=	10,400*	.0720632	=	749.42
S(3,3)	=	7,800*	.0024987	=	19.42
S(3,4)	=	7,800*	.0199032	=	155.22
S(4,4)	=	29,400*	.0024987	=	73.206

i MONTH	RETURNS Z(i)	SALES L(i)
2	R(2) = 48	S(1,2) + S(2,2) 720.38 + 25.896 = 746.482
3	R(3) = 173	S(1,3) + S(2,3) + S(3,3) 2608.57 + 206.96 + 19.42 = 2835.17
4	R(4) = 372	S(1,4) + S(2,4) + S(3,4) + S(4,4) 5667.11 + 749.42 + 155.22 + 73.2 = 6645.31

$$r(4) = \frac{\sum_{i=2}^{4} (Ri)}{\sum_{i=2}^{4} \sum_{p=1}^{i} S(p,i)} = \frac{R(2)+R(3)+R(4)}{S(1,2)+S(2,2)+S(1,3)+S(2,3)+S(3,3)+(1,4)+S(2,4)+S(3,4)+S(4,4)} =$$

$$\frac{98 + 173 + 372}{746.48 + 2835.17 + 6645.31} = .0629$$

Variance of the return rate Vr(x) =

$$\frac{\sum_{i=2}^{x}\left[d(i)^2/L(i)\right]}{\left[(x-1)-2\right]*\sum_{i=2}^{x}L(i)}$$

Where d(i) = Z(i) - r(x)*L(i)

for x=4

i	Zi	L(i)	r(x)L(i)	Z(i)-r(x)L(i)	$d(i)^2$
2	48	746.482	43.284	4.716	22.2406
3	173	2835.17	164.394	8.60551	74.0548
4	372	6645.31	385.322	-13.3215	177.462
	Σ	10226.9			

i	$\frac{d(i)^2}{L(i)}$
2	.02979
3	.02612
4	.0267
Σ	.08261

$$Vr4 = \frac{.08261}{1*10226.9} = 8.078 \times 10^{-6}$$

The developed computer program performs repeated calculations for each monthly period of paired data, in a fashion similar to the sample calculation. For each of three data sets containing twenty-five pairs of sales and return data, 144 sets of f(w) values were used to calculate the return rate r(x) and Vr(x) corresponding to each incremental month of paired data. The 144 sets of f(w) were generated by using values for (a) from .5 to 6 in increments of .5, and values of (b) from .5 to 6 in increments of .5 as constants in the Beta equation previously discussed. This range of (a) and (b) adequately covered the range of f(w) needed to generate a response surface for Vr(x) which indeed had an adequate minimum value for each incremental month of sales and return data analyzed.

The following three tables list for each incremental month the minimum variance Vr(x) and the corresponding return rate r(x) and (a) and (b) values which define the f(w).[3] Additionally three graphs follow which illustrate the return rate using weighted sales.

Weighted Sales = S(p,y) = S(p)*f(y-p+1)

where p = month of sale

y = month of return

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3 Appendix A, Appendix B, Appendix C contains the raw data for Data Set 1, Data Set 2, and Data Set 3 used in the calculations.

corresponding to the minimum $Vr(x)$. Also, on the graph of return rate is another plot using non-weighted sales for a comparison.

Return rate using non-weighted sales:

$$\text{Return rate } C(x) = \frac{\sum\limits_{M=1} R(M)}{\sum\limits_{M=1} S(M)}$$

TABLE 3

DATA SET Ø1

MONTH	RETURN RATE %	VARIANCE x 10-9	BETA PARAMETERS A	B
4	7.23	330	5	5.5
5	1.22	13330	.5	6
6	2.04	69982	.5	6
7	2.83	68594	1.5	6
8	4.87	45577	4	6
9	4.28	32110	3.5	6
10	3.21	25183	1.5	6
11	3.18	18365	1.5	6
12	4.83	14536	4	6
13	4.94	11623	4	6
14	5.41	9890	4.5	6
15	5.90	9946	5	6
16	5.91	8033	5	6
17	5.95	6788	5	6
18	6.35	8757	5	5.5
19	6.76	10466	4	4
20	6.76	9081	4	4
21	6.75	8000	4	4
22	6.86	8174	4	4
23	6.83	7407	4	4
24	6.82	6735	4.5	4.5
25	6.89	6661	4	4

TABLE 4

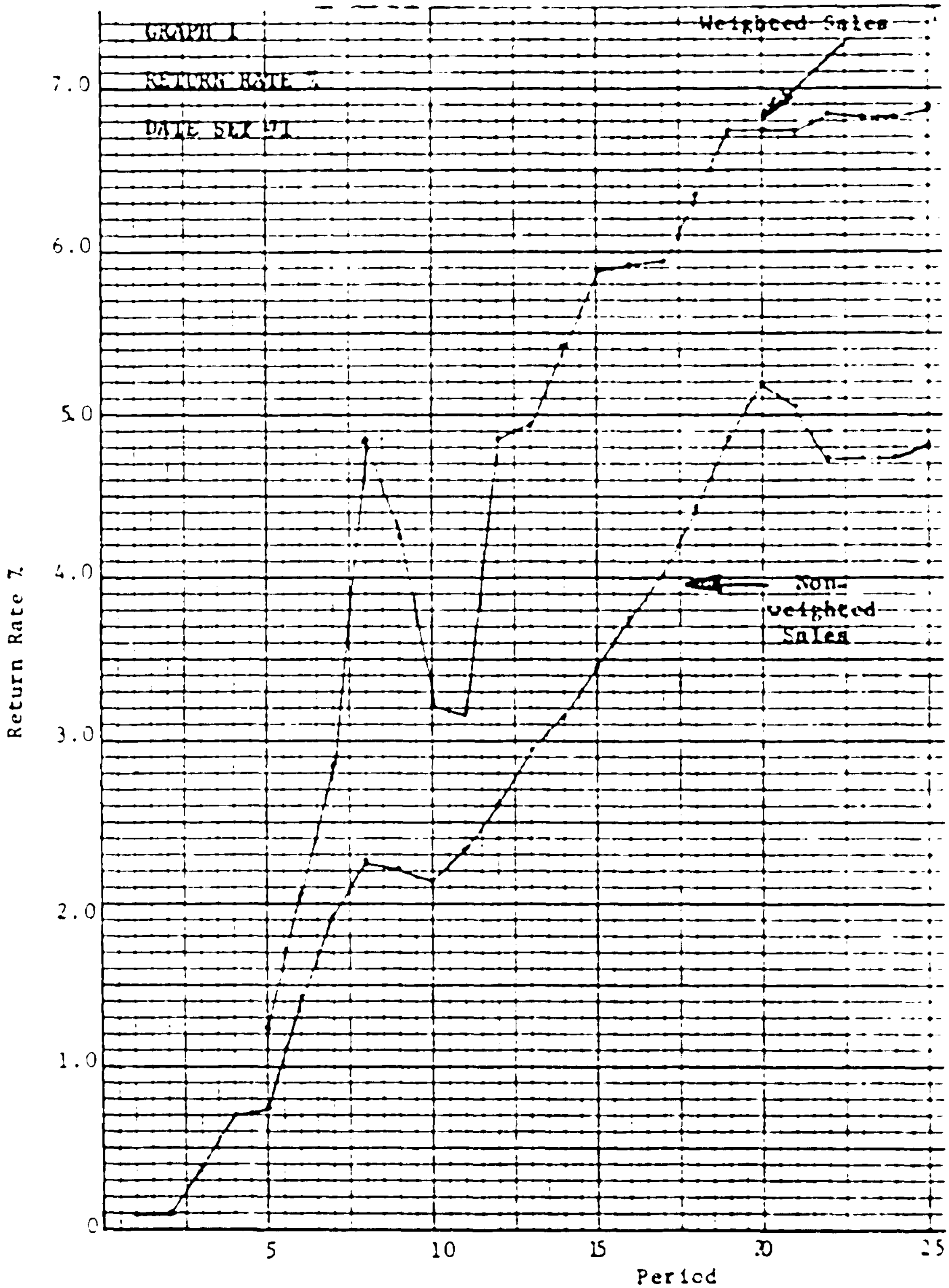
DATA SET #2

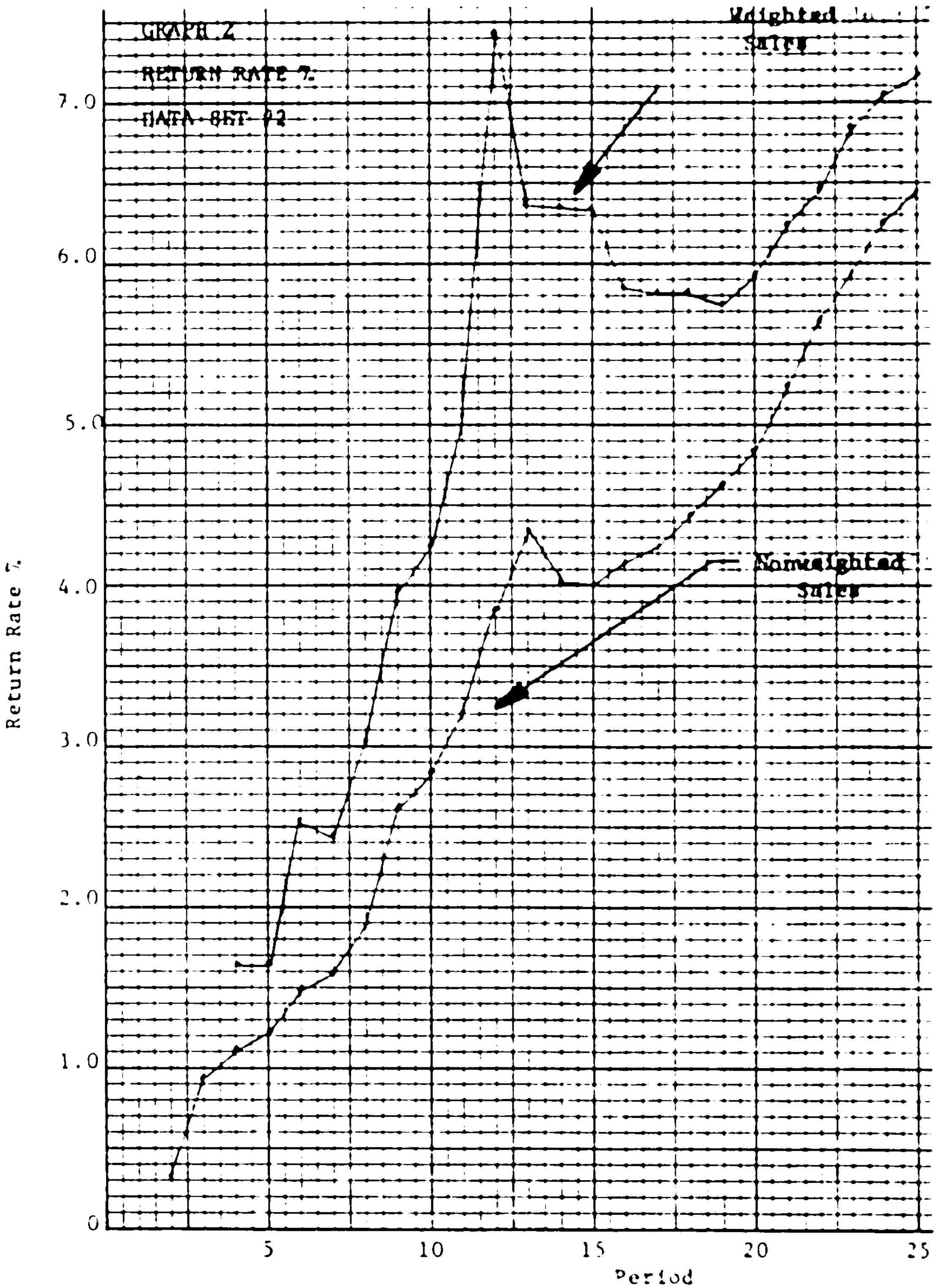
MONTH	RETURN RATE %	VARIANCE x 10-9	BETA PARAMETERS A	B
4	1.62	27935	0.5	6
5	1.68	8740	0.5	6
6	2.54	8002	1.5	6
7	2.46	4705	1.5	6
8	3.04	5958	2.0	6
9	3.97	21373	2.5	6
10	4.24	15505	2.5	5.5
11	5.06	15837	2.0	3.5
12	7.42	25642	1.5	1.5
13	6.39	20333	1.5	2.0
14	6.37	16738	2	2.5
15	6.33	13866	2	2.5
16	5.87	12822	2	3.0
17	5.82	11082	2	3.0
18	5.82	9543	2	3.0
19	5.76	8625	2	3.0
20	5.92	9776	2	3.0
21	6.23	9954	2	2.5
22	6.48	15014	2	2.5
23	6.85	15927	2	2.0
24	7.06	19729	2	2.0
25	7.19	19255	1.5	1.5

TABLE 5

DATA SET #3

MONTH	RETURN RATE %	VARIANCE x 10-9	BETA PARAMETERS A	B
4	1.08	8497	.5	6
5	.79	3991	.5	6
6	.77	1698	.5	6
7	1.35	6846	1.5	6
8	2.26	10643	2.5	6
9	2.33	7061	2.5	6
10	2.16	6369	2.5	6
11	2.15	4532	2.5	6
12	2.11	3462	2.5	6
13	2.11	2674	2.5	6
14	2.06	2244	2.5	6
15	2.02	1902	2.5	6
16	2.09	1836	2.5	6
17	2.17	1879	2.5	6
18	2.25	2006	2.5	6
19	2.33	2073	2.5	6
20	2.33	1763	2.5	6
21	2.29	1610	2.5	6
22	2.18	1993	2.5	6
23	2.01	1952	2	6
24	1.97	1826	2	6
25	1.88	1705	1.5	6

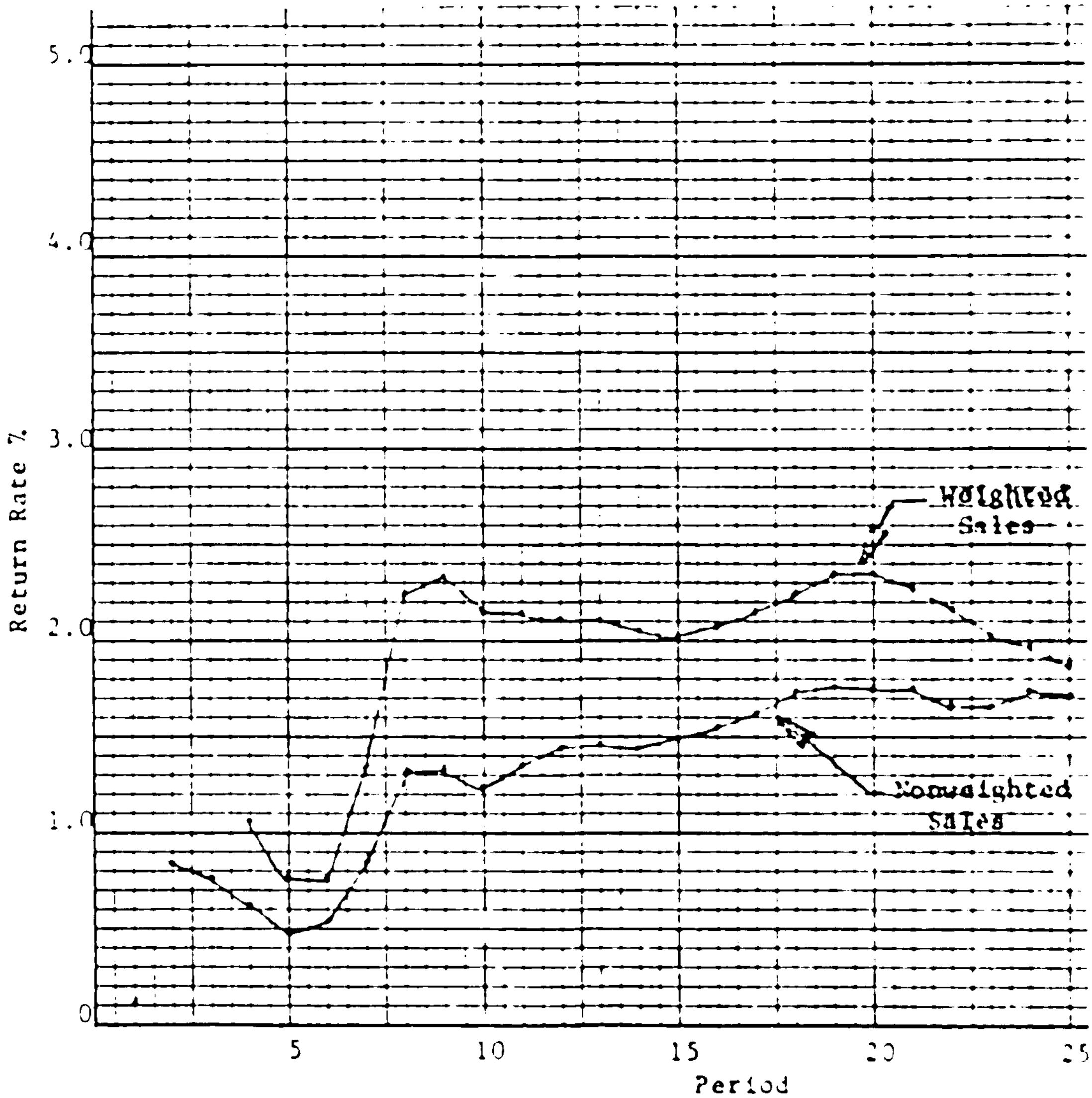

GRAPH I
RETURN RATE %
DATE SEP '71
Weighted Sales
Non-weighted Sales
Return Rate %
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
5
10
15
20
25
Period


GRAPH 2
RETURN RATE %
DATA SET #2
Weighted
Sales
Nonweighted
Sales
Return Rate %
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
5
10
15
20
25
Period

GRAPH 3

RETURN RATE %

DATA SET #3



CONCLUSIONS

The reason for using a sales weighting scheme is to develop a means whereby return rate estimates can be arrived at quickly and accurately. If the return rate was calculated by taking a ratio of cumulative returns to cumulative sales, without factoring, the true return rate would be unknown for a long period of time. The return rate would be grossly underestimated early by at least one order of magnitude but would eventually converge on the true value. Likewise if the improper weights are chosen, in other words a random selection of weights, without regard to minimizing the variance of the return rate then the return rate estimate would also be in error. Since random weights can produce an inifinite variety of return rates a decision rule, such as minimum variance is necessary to obtain the best return rate.

The Beta model appears to represent the realistic customer return patterns associated with the products under consideration. The basic shapes of the Beta Model for the various Data Sets are illustrated in the appendix.[4] The best fit a's and b's indicate that a delay of a number of months usually exists between sale and receipt of product return information. Signifying in reality a true delay in the receipt of product return information at the factory. Also the returns from a given month of sales are not

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[4] See Appendix D and Appendix E.

uniformly distributed throughout the warranty period. The returns tend to be tightly distributed centering near the middle of the warranty period.

Again this signifies a normal happening in the way these particular products fail. Most of the products, within the warranty period, which are returned for credit or repair have problems associated with initial failures. If the product is going to be returned during the warranty period, the decision is made rather quickly by the consumer; either before the first use or after the first several product uses. Very seldom will component wearout or product wearout enter the warranty return picture.

In summary this paper shows the development of a Return Rate estimating system which uses the aid of a computer to calculate the optimum value for sales weights such that the estimated values of returns closely approximates the true number of warranty returns within eight months from the time of first sale.

FOR FUTURE INVESTIGATION

A rather cumbersome manual system of searching for optimum f(v) values was used during the development of this thesis. An automatic searching system to find the optimum values of f(v) efficiently would make this routine more practical for computerized return rate calculations and reporting.

Also a method for detecting changes in the return rate would be beneficial as the number of months of data grow. This model is cumulative and develops a good deal of inertia as the months accumulate. Therefore, changes in return rate will tend to become difficult to detect. A method of accomplishing this item would include rolling the data once the optimum number of months of sales and return data were in the calculating base. For example once the optimum month of data were derived, the subsequent calculations would contain an added months worth of data and the oldest months worth of data would be discarded.

APPENDIX A

TABLE 6

DATA SET Ø1

MONTH	RETURNS R(m)	Σ RETURNS	SALES S(m)	Σ SALES	$\frac{\Sigma \text{RETURNS}}{\Sigma \text{SALES}}$
1	3	3	36200	36200	.01
2	48	51	10400	46600	.11
3	173	224	7800	54400	.41
4	372	596	29400	83800	.71
5	302	898	33800	117600	.76
6	1118	2016	24100	141700	1.42
7	1094	3110	20300	162000	1.92
8	1052	4162	21700	183700	2.27
9	914	5076	45800	229500	2.21
10	1535	6611	78000	307500	2.15
11	1267	7878	27300	334800	2.35
12	1764	9642	34600	369400	2.61
13	2138	11780	33300	402700	2.93
14	2470	14250	46200	448900	3.17
15	2898	17148	44600	493500	3.47
16	2474	19622	30200	523700	3.75
17	2592	22214	29100	552800	4.02
18	3272	25486	23100	575900	4.43
19	3389	28875	17200	593100	4.87
20	2428	31303	8900	602000	5.20
21	2240	33543	61300	663300	5.06
22	2710	36253	100900	764200	4.74
23	1961	38214	41100	805300	4.75
24	2055	40269	40700	846000	4.76
25	3058	43327	52700	898700	4.82

APPENDIX B

TABLE 7

DATA SET Ø2

MONTH	RETURNS R(m)	Σ RETURNS	SALES S(m)	Σ SALES	Σ RETURNS / Σ SALES
1	0	0	400	400	.00
2	29	29	9100	9500	.31
3	168	197	11600	21100	.93
4	190	387	13100	34200	1.13
5	272	659	19700	53900	1.22
6	425	1084	17800	71700	1.51
7	369	1453	18700	90400	1.61
8	583	2036	16500	106900	1.90
9	903	2939	5900	112800	2.61
10	672	3611	14700	127500	2.83
11	887	4498	12300	139800	3.22
12	1193	5691	7200	147000	3.87
13	814	6505	3600	150600	4.32
14	776	7281	29900	180500	4.03
15	811	8092	21400	201900	4.01
16	642	8734	8800	210700	4.15
17	731	9465	12600	223300	4.24
18	799	10264	8300	231600	4.43
19	647	10911	4700	236300	4.62
20	1024	11935	9100	245400	4.86
21	897	12832	0	245400	5.23
22	1190	14022	3600	249000	5.63
23	927	14949	3600	252600	5.92
24	1101	16050	2500	255100	6.29
25	778	16828	5800	260900	6.45

APPENDIX C

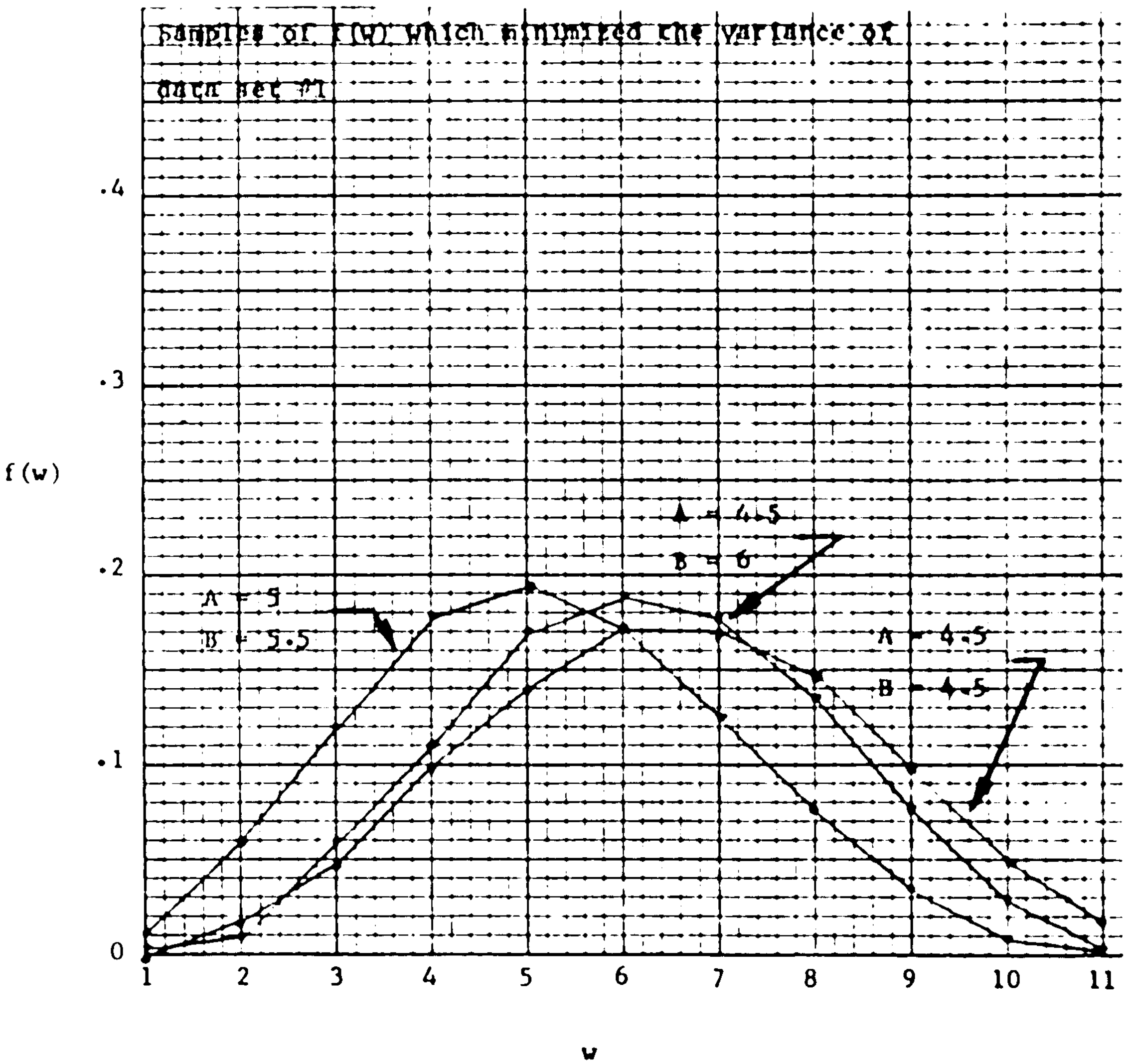
TABLE 8

DATA SET #3

MONTH	RETURNS R(m)	$\sum$ RETURNS	SALES S(m)	$\sum$ SALES	$\frac{\sum \text{RETURNS}}{\sum \text{SALES}}$
1	3	3	2800	2800	.11
2	37	40	2000	4800	.83
3	62	102	8300	13100	.78
4	201	303	35400	48500	.62
5	302	605	74700	123200	.49
6	330	935	42600	165800	.56
7	746	1681	27400	193200	.87
8	1009	2690	11800	205000	1.31
9	920	3610	60500	265500	1.36
10	621	4231	76100	341600	1.24
11	958	5189	40000	381600	1.36
12	887	6076	35300	416900	1.46
13	992	7068	60200	477100	1.48
14	887	7955	67900	545000	1.46
15	911	8866	44800	589800	1.50
16	1453	10319	76700	666500	1.55
17	1679	11998	76300	742800	1.62
18	1868	13866	51900	794700	1.74
19	2045	15911	98500	893200	1.78
20	1675	17586	96100	989300	1.78
21	1480	19066	94300	1083600	1.76
22	962	20028	107900	1191500	1.68
23	1185	21213	70000	1261500	1.68
24	1129	22342	22600	1284100	1.74
25	1043	23385	77000	1361100	1.72

GRAPH 4

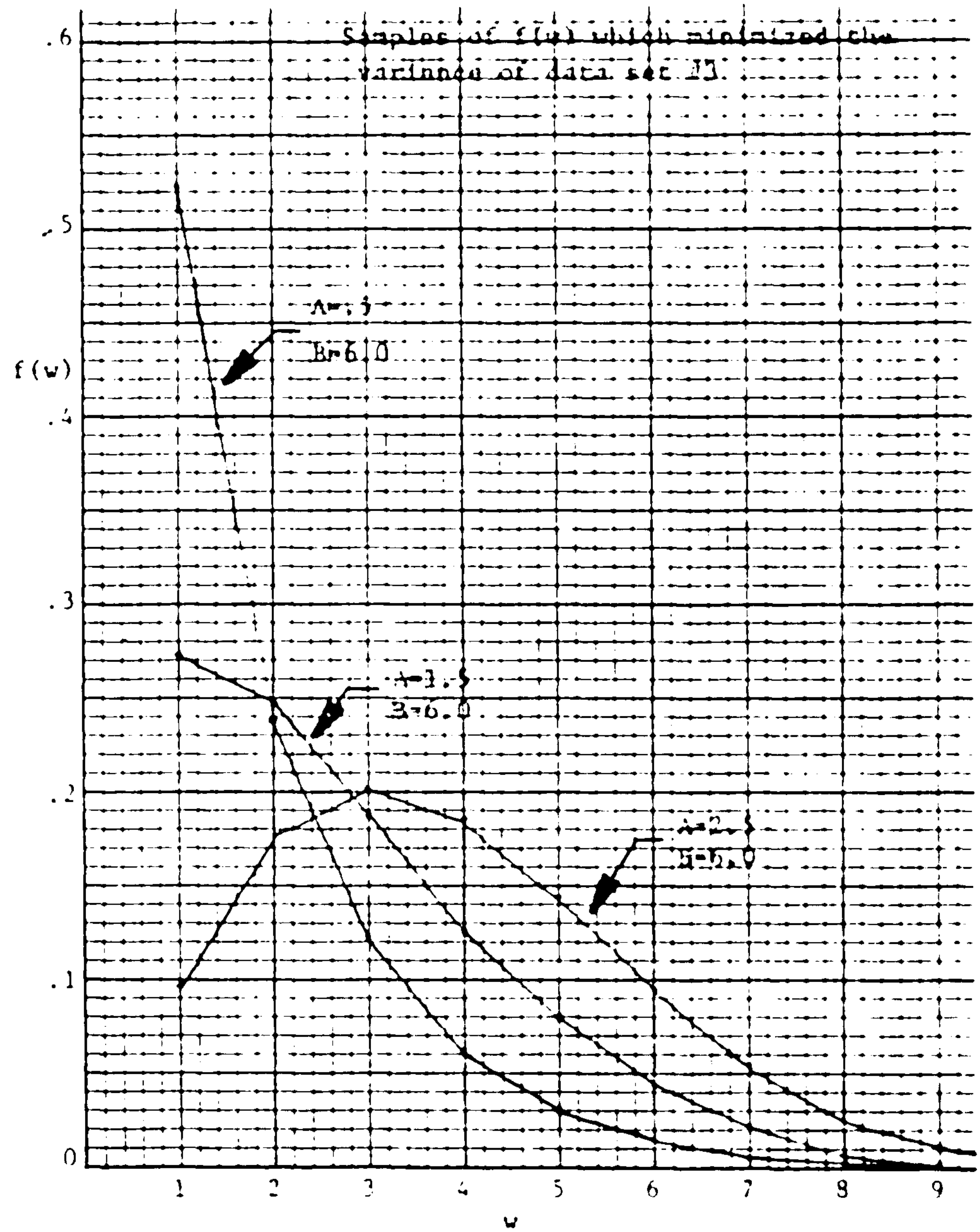
APPENDIX D



w = monthly period after the sale

GRAPH 5

APPENDIX E



w = monthly period after the sale

THE "VITA"

Name	- William Terry Lease
Place of Birth	- Swoyersville, Pennsylvania
Date of Birth	- November 11, 1944
Name of Mother	- Margaret (Hardish) Lease
Name of Father	- Benjamin Lease
Name of Spouse	- Patricia O. Lease
Name of Children	- Benjamin Neal Lease (age 10)
	- William Terry Lease, Jr. (age 4)

Institutions Attended

School	Curriculum	Dates Attended	Degree
Swoyersville H.S.	Academic	9/58-6/62	Diploma
Wilkes College	Engineering	9/62-7/64	Certificate
Penn State University	Electrical Engineering	9/64-6/66	Bachelor of Science

Work Experience

Company	Title	Dates
General Electric	Final Product Engineer and Test Lab Supervisor	7/69-10/73
General Electric	Manager, Quality and Reliability Engineering	10/73-10/74
General Electric	Manager, Process Control Engineering	10/74-Present

BIBLIOGRAPHY

Bazovsky, Igor. Reliability Theory and Practice. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1963.

Draper, N. R. and H. Smith, Applied Regression Analysis. New York: John Wiley and Sons, 1966.

Hahn, Gerald J. and Samuel S. Shapiro, Statistical Models in Engineering. New York: John Wiley and Sons, 1968.

Mandel, John. The Statistical Analysis of Experimental Data. New York: John Wiley and Sons, 1964.

Shooman, Martin L. Probabilistic Reliability: An Engineering Approach. New York: McGraw-Hill, 1968